IMMISCIBLE FLUID DISPLACEMENT IN NONLINEAR SEEPAGE FLOW

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Abstract: The existing theory of immiscible fluid flow is extended to the seepage with nonzero initial shear stress of viscoplastic media. An analog of the Buckley-Leverett frontal-displacement theory is constructed.

1. In the theory of motion of multiphase fluids [1-3] it is assumed that each phase moves separately in accordance with Darcy's law: the seepage velocity of each phase is assumed to be proportional to the pressure gradient and the effective permeability depends on the distribution of pore volume between phases (saturation). Numerous investigations have shown that this schematization is a fruitful one and allows for accurate description of the main features of the multiphase flow processes.*

Below, the existing theory is extended to the case for which one or both of the fluids do not obey Darcy's law. In itself, the nonlinearity may be associated either with the truly non-Newtonian behavior of the fluid (e.g., a number of natural oils [4,5], emulsions [6], foams, etc.) or with the particularly strong nonviscous interaction between the fluid and the rock (e.g., water in argillized sandstone [7]). In all these cases, the nonlinearity is manifested in the region of low seepage velocities. This is important to note, since the basic problem of the theory of two-phase motion is the displacement problem describing the displacement of oil by water during the course of flooding. In this case, the low-velocity region is most important, since it covers most of the bed. The discussion that follows is based on Efros' thorough analysis of the processes associated with displacement and two-phase flow [8].

The simplest model of a fluid for which the flow law is nonlinear in the low-velocity region is the model of a viscoplastic medium, in particular, the Bingham-Schwedoff model used below. In this case, the relation between the stresses τ and the shear rate du/dn has the form

$$\tau = -\tau_0 - \mu du / dn \quad (du / dn > 0),$$
$$|\tau| \leqslant \tau_0 \quad (du / dn = 0), \tag{1.1}$$

where τ_0 is the critical value of the shear stress and μ is viscosity.

2. Nonlinear flow of two-phase systems. First of all, we will show that the theory of multiphase flow remains essentially valid for non-Newtonian media. The nucleus of the theory is the assumption that, at any point in the flow region, the phase distribution in the pore space is determined by capillary forces and does not depend on the hydrodynamic forces associated with motion of the fluid (regarding the limits of applicablitiy of this assumption see, for example, [3.8]). In this case, the process of occupation of the pore space is assumed to be unidirectional (e.g., the gradual replacement of oil by water), the fluid with the greater wetting properties tending to occupy the smaller pores. At first glance, if the fluid is viscoplastic and has a nonzero initial shear stress τ_j (the subscript denotes the phase number), the phase distribution will change; a plastic fluid will not be able to penetrate into the narrowest pore channels, just as displacement from such pores will be difficult. It is easy to see, however, that this is not so.

Let a fluid with critical shear stress τ_2 be displaced from a porous medium. Here τ_2 is the true critical shear stress; i.e., at stresses less than τ_2, there is no motion of the fluid at all. The displacing fluid has a shear stress τ_1. We denote the surface tension at the phase interface by σ; at the interface there exists a certain capillary pressure jump $p_c = |p_1 - p_2|$, where $p_1 < p_2$ if $\sigma > 0$, and the fluid with the greater wetting power displaces the less-wetting one. Other things being equal, p_c is determined by the mean radius of curvature of the phase interface and hence [1-3] depends on the saturation s, $p_c = p_c(s)$.

We assume that the volume initially occupied by the second fluid is subsequently partially filled (to saturation s) by the first fluid, which, to be specific, is assumed to have the greater wetting power. We will consider the nature of the distribution of both phases over the pores. First of all, it is clear that the stresses τ_i must be very small, since, otherwise, under actual conditions, there can be no question of any displacement of fluid. In fact, if we denote the characteristic value of the externally applied pressure head by Δp, the corresponding distance (macroscale) by L, and the characteristic dimension of the pores (internal scale) by d,† obviously

$$\omega \Delta p / L > \tau_i / d \quad (i = 1,2),$$

where the coefficient ω is on the order of unity. As usual, we will consider only elements of the porous medium which are large compared with the inside dimension d. If the characteristic dimension of such a macrovolume element is D, the corresponding pressure drop is of the order of $\Delta p D / L$. If it is assumed that the phase distribution differs significantly from that corresponding to local capillary equilibrium, a capillary pressure jump equal in order of magnitude to σ/d will bring the phases into equilibrium. If d is sufficiently small,

$$D \ll \sigma L / (\Delta p d); \tag{2.2}$$

the equilibrium phase distribution will be established so quickly that in solving hydrodynamic problems it may be assumed to be an equilibrium distribution all the time. In view of the condition that $d \ll D$, it is necessary that

$$d^2 < \sigma L / \Delta p < \alpha \sigma d / \tau_i \tag{2.3}$$

(the latter in view of (2.1)), which is usually satisfied in oil field work. Consequently, with the same conditions

$$\tau_i \ll \sigma / d \tag{2.4}$$

and it is also possible to neglect the effect of the non-Newtonian properties of the fluid on phase redistribution. In other words, the

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* In itself, this assumption, whether explicit or implicit, is associated with the fact that, in any volume element, the pressure created by capillary forces on departure from equilibrium considerably exceed the pressure drop due to hydrodynamic forces. Thus, the hydrodynamic forces are capable of evoking only a slight change in the phase distribution within the pore space on the scale of the individual element. On the other hand, in terms of the whole bed, the direct effect of the pressure drop created by capillary forces is negligible.

If for some reason, the phase distribution in the porous medium is a non-equilibrium distribution, the departure from equilibrium will be smoothed out differently depending on the space scale. Distrubances in the volume element are smoothed out by capillary forces. Disturbances involving the entire bed smoothed out by displacement of the bed by the hydrodynamic head (the fluid velocities at any point depend on the local saturation, which may ready be assumed to be in equilibrium: precisely this case is considered below). Finally, there is an intermediate scale on which the effects of capillary and hydrodynamic forces must be jointly taken into account.

† The quantity $\sqrt{k/m}$, where k is the permeability and m the porosity of the medium, is often taken as the internal scale. Then, for media of the same type $d = C\sqrt{k/m}$ and the proportionality factor C may be the order of several tens.

phases are distributed in accordance with the conditions of capillary equilibrium, just as if there were no resistance forces at all; at any moment each of the phases moves in its own volume of the pore space, just as if that volume were the pore space of some homogeneous medium and the other phase were completely absent.

As the saturation changes, the properties of these fictitious" porous medium with the first, wetting phase. Assuming that, initially, the wetting phase preferentially fills the smaller pores, we arrive at the conclusion that, qualitatively, the $d_i(s)$ curve takes the form shown in Fig. 1, where d denotes the pore size averaged over the entire pore volume. From the condition of independent phase motion it follows that the seepage velocity of a given phase is determined by its properties, by the pressure gradient, and by the geometry of the volume it occupies, but not by the porperties of the other phase. The properties of a viscoplastic medium are characterized by the viscosity μ and the initial shear stress τ_i, and the geometry of the occupied volume by the characteristic pore size $d_i(s)$, so that

$$\mathbf{u}_i = -\frac{d_i^2(s)}{\mu_i} \operatorname{grad} p f\left(\frac{\tau_i}{d_i |\operatorname{grad} p|}\right). \tag{2.5}$$

Assuming that the function f^* can be expanded, we obtain:

$$\mathbf{u}_i = -C \frac{d_i^2}{\mu_i} \operatorname{grad} p \left(1 - B \frac{\tau_i}{d_i |\operatorname{grad} p|}\right). \tag{2.6}$$

This analysis presupposes that the phase in question continuously fills the space it occupies, i.e., the phase is connected. The connectivity condition is satisfied when the porous medium with the given phase is highly saturated, but when the saturation falls, the fraction corresponding to isolated droplets gradually increases until finally, at a certain saturation, the entire phase is divided into individual droplets and motion ceases. On the phase-permeability curves this corresponds to the vanishing of the corresponding phase permeabilities.

With this in mind, we rewrite (2.6) in the form

$$\mathbf{u}_i = -\frac{k_i(s)}{\mu_i} \operatorname{grad} p \left(1 - B \frac{\tau_i}{d_i |\operatorname{grad} p|}\right)$$
$$k_i(s) \equiv k f_i(s) \leqslant C d_i^2(s). \tag{2.7}$$

Here, in view of the conclusions of section 1, the phase permeabilities $k_i(s)$ depend on the saturation s in the same way as the phase permeabilities for ordinary Newtonian fluids.

It is more convenient to rewrite (2.7) in the form

$$\mathbf{u}_i = -\frac{k}{\mu_i} f_i(s) \left(\operatorname{grad} p - \gamma_i \frac{\operatorname{grad} p}{|\operatorname{grad} p|}\right). \tag{2.8}$$

Here it is assumed that when $|\mathbf{u}_i| > 0$ the first term is greater in absolute value than the second; otherwise $\mathbf{u}_i = 0$. Relation (2.8) expresses the extension to two-phase flow of the law of seepage, with an initial gradient used to describe the seepage of viscoplastic fluids in [9,10] and the motion of water through argillized porous media in [11]. In this case, the limiting gradients $\gamma_i = B\tau_i / d_i$ of the corresponding phases are functions of saturation.

Relations (2.7) and (2.8) represent the required extension of the basic relations of two-phase flow to the motion of viscoplastic fluids. In this case, it is also necesary to determine the dimensionless constant B and the functon $d_i(s)$. Like the relations of two-phase flow theory for viscous fluids, expression (2.8) cease to apply in regions where the saturation experiences sharp changes.
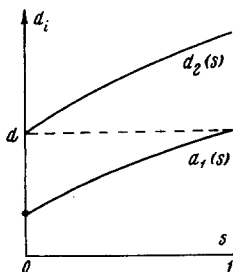

Fig. 1

Such regions correspond to the displacement front; near the displacement front, the capillary and hydrodynamic forces are comparable in magnitude, inequality (2.2) is violated, and it is no longer possible to assume that the phases will move without interacting. In the region of the front, the very concept of phase permeabilities becomes questionable (this has been noted by G. I. Barenblatt), since the nature of phase motion is still unclear. In what follows, region of sharp change of saturation will be excluded from consideration.

3. Frontal displacement of non-Newtonian fluids. Buckley-Leverett theory. We will consider the simple problem of one-dimensional frontal displacement of non-Newtonian fluids in the Buckley-Leverett formulation [1-3]. Assuming that both fluids are incompressible, we have

$$u_1 + u_2 = u(t) \tag{3.1}$$

where u (t) is a known function of time. We neglect the pressure difference between the fluids and set

$$p_1 = p_2 = p(x, t). \tag{3.2}$$

From (2.8), written for each of the phase, we have

$$\frac{u_1\mu_1 + Bkf_1(s)\tau_1/d_1}{kf_1(s)} = \frac{u_2\mu_2 + Bkf_2(s)\tau_2/d_2}{kf_2(s)} = -\frac{\partial p}{\partial x}.$$

On the other hand, adding Eqs. (2.8), we obtain (assuming that $u > 0$, $\partial p/\partial x < 0$)

$$u = u_1 + u_2 =$$
$$= -\left(\frac{kf_1(s)}{\mu_1} + \frac{kf_2(s)}{\mu_2}\right)\frac{\partial p}{\partial x} - \frac{kf_1(s)\gamma_1}{\mu_1} - \frac{kf_2(s)\gamma_2}{\mu_2}.$$

Forming the expression for $\partial p/\partial x$ and substituting into the previous relations, we obtain

$$u_1 = u \frac{f_1(s)}{f_1(s) + \mu f_2(s)} + \frac{f_1(s)f_2(s)}{\mu_2[f_1(s) + \mu f_2(s)]}(\gamma_2 - \gamma_1);$$
$$\mu = \frac{\mu_1}{\mu_2} \tag{3.3}$$

and the analogous relation for u_2 (obtained by interchanging subscripts). Expression (3.3) is an analog of the usual Buckley-Leverett relation, and the coefficient of u on the right-hand side is the Buckley-Leverett function. Expression (3.3) ceases to be valid if its right-hand side becomes negative; in this case, it is necessary to set $u_1 = 0$. However, if it exceeds u, it is necessary to set $u_1 = u$. This remark also applies to u_2. We rewrite (3.3) in the form

$$u_1 = uF(s, \mu, u) \tag{3.4}$$

and substitute it into the continuity equation

$$m \frac{\partial s}{\partial t} + \frac{\partial u_1}{\partial x} = 0. \tag{3.5}$$

In this case, for the saturation s, we obtain the usual equation of the Buckley-Leverett theory; the properties of its solutions are determined by the form of the function F.

Considering the physical significance of the parameters that have been introduced, we see that with fairly natural assumptions the function F remains monotonic with respect to the saturation s. In fact, it is easy to obtain the relation

$$\frac{u_1 + kf_1\gamma_1/\mu_1}{u - u_1 + kf_2\gamma_2/\mu_2} = \frac{f_1}{\mu f_2} \tag{3.6}$$

which we can represent in the form

$$\frac{u_1\mu_1(kf_1\gamma_2)^{-1} + \gamma_1/\gamma_2}{(u - u_1)\mu_2(kf_2\gamma_2)^{-1} + 1} = 1. \tag{3.7}$$

We assume that the ratio d_2/d_1 decreases with increase in s, which is consistent with the form of the curves in Fig. 1. Moreover, f_1 must

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* Generally speaking, the saturation s should also be included as an independent argument of the function f. Accordingly, the quantity B in (2.6) is found to depend on s.
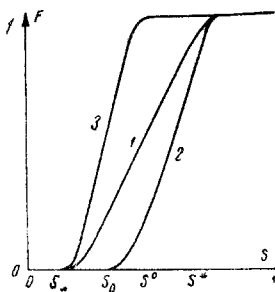
Fig. 2

increase with s more rapidly than d_1 and hence more rapidly than d_2. Then, for fixed u and u_1 the expression on the left-hand side of (3.7) decreases with an increase in s and, at fixed s, increases with an increase in u_1. Thus, we find that u_1 increases with an increase in s.

At the same time, in the general case, it is impossible to assert that the curve F (s) has only one point of inflection, as in the case of Newtonian fluids. In this respect the flow of viscoplastic fluids is similar to the displacement in the mass force field examined by Buzinov and Charnyi [12] (see also [2]).

The appearance of an additional term in the expression for u_1 (Eq. (3.3)) may lead to an important change in the nature of the displacement. Let us start by assuming that the difference $\gamma_1 - \gamma_2$ is positive. Then, at sufficiently small velocities u, expression (3.3) becomes negative for all saturations s between s^* and some value of s_0. Accordingly, the function F will have the form of curve 2 in Fig. 2 (curve 1 corresponds to $\tau_1 = \tau_2 = 0$).

The quantity s_0 is the minimum saturation value for the displacing phase. As the difference $\gamma_1 - \gamma_2$ increases, and the displacement velocity u decreases, this quantity increases to a certain limiting value s^*, which represents the maximum possible value of saturation for the displacing phase. In other words, in the case of slow displacement by a fluid having a sufficiently large initial shear stress, the displacement tends to the piston case with maxium completeness of displacement. This determines the possibilities of displacement with viscoplastic fluids. (This is fully consistent with the actual intensification of displacement by means of foams for which a nonzero initial shear stress is observed.)

As may easily be seen from the above relations, in our case the increased efficiency of displacement is associated with the fact that at low velocities the ratio of the pressure gradients in the displacing and displaced fluids becomes infinitely large. In principle, the same effect can be obtained by increasing the viscosity of the displacing fluid in the ordinary Buckley-Leverett theory.

However, it is important that as the viscosity increases the pressure losses near injection wells increase substantially, whereas the existence of a limiting gradient adds only slightly to the local pressure losses near the wells.

On the other hand, when the opposite relation $\gamma_1 < \gamma_2$ is satisfied at sufficiently small u, the F(s) curve has the form of curve 3 in Fig. 2; the displacement efficiency falls the more sharply, the smaller the velocity u.

Note added in proof. A recent experimental study [13] gives data on the dependence of the phase permeabilities on critical shear stress (for displacement of a model viscoplastic fluid by a gas). We note two points that may account for the disagreement between the conclusions reached above and these experimental data. First of all, it was assumed above that γ is on the order of the mean pressure gradient in the bed: $\gamma \sim 10^{-1}$ at/m = 10^3 dyne/cm^3. For a mean pore side d $\sim$ $\sim 10^{-2}-10^{-3}$ cm this corresponds to an initial shear stress $\tau \sim 1-10$ dyne/cm^2, which is much less than the shear stress for the fluids used in the experiment. Secondly, it should be borne in mind that the relative phase permeabilities introduced above no longer represent the ratio of the fluid flow rate in two-phase flow to the flow rate in single-phase flow with the same pressure gradient. In the experiment, however, they were apparently determined in precisely this way. Finally, we note that since a viscoplastic fluid is more wetting than a gas, in Fig. 1, curve 1 should be assigned to the fluid.

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